American University of Beirut

MATH 202

Differential Equations

Spring 2014

Quiz #2

Time: 70 minutes

Exercise 1 (22 pts)

Find the right form of yp **-do not find the constants-** for each of the following differential equations

a) $y''' - 3y'' + 4y' = 1 + xe^x + 2e^x \sin x$

b) $y^{(5)} + 27y'' = 6x + x^2 \sin(3x)$

Exercise 2 (20 pts)

Solve the differential equation: $y'' - 2y' + 2y = e^x \tan x$

Exercise 3 (10 pts)

Give the general solution of the differential equation: $x^2y''' - 3xy'' + y' = 0$

Exercise 4 (16 pts)

a) Prove that the reduction of order in general for $y'' + p(x)\, y' + q(x)\, y = 0$ via the substitution ***$y=y_1u(x)$***, where y_1 is a non-zero solution of the homogeneous equation gives

$$u'' + \left(p(x) + \frac{2y_1'}{y_1}\right)u' = 0$$

b) Use part a) to find the general solution of the differential equation (E) given that $y=x^3$ is a solution

(E): $(x^3 - x^2)\, y'' + 2(-x^2 + 2x)\, y' - 6y = 0$

Exercise 5 (24 pts)

Consider the differential equation: (E): $(x^2 - 1)\, y'' + xy' - y = 0$

Let $y = \sum_{n=0}^{+\infty} c_n x^n$ be a series solution for (E) about the ordinary point x = 0

a) What would be the radius of convergence of the above series? Justify your answer

b) Find two series solution for the equation (E)

c) Use reduction of order to express the series found in part b) in terms of an integral of the usual functions

Exercise 6 (8 pts)

Use the substitution $2x + 1 = e^t$ to transform the given differential into equation with constant coefficients and then **STOP**

$$\frac{1}{4}(2x + 1)^2y'' - \frac{5}{4}(4x - 2)\, y' + 10y = \ln(2x+1)$$